NONLINEAR PLASMA THEORY

R. Z. SAGDEEV and A. A. GALEEV
Novosibirsk State University, U.S.S.R.

Revised and Edited by

T. M. O'NEIL
University of California, San Diego

D. L. BOOK
Lawrence Radiation Laboratory

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T. M. O'Neil
D. L. Book

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Introduction

An essential simplification of linear plasma theory derives from the fact that arbitrary perturbations can be expressed as a superposition of eigenmodes, with each eigenmode evolving independently of the others. In this book, we extend this systematic approach to weakly nonlinear plasmas. Arbitrary perturbations are still expressed as a superposition of linear eigenmodes, but the nonlinearity provides a weak interaction between the modes. Consequently, the coefficients in the superposition of modes become slowly varying functions of time and eventually assume values quite different from those predicted by linear theory.

This approach to nonlinear plasma theory is usually referred to as the theory of weak turbulence. It can be justified by a perturbation expansion of the Vlasov (or fluid) equation when the energy in the excited spectrum of modes is small compared with the total plasma energy. Of course, the energy in the excited spectrum must be larger than thermal noise to be of any interest. In other words, the theory of weak turbulence can appropriately describe the evolution of an initially unstable plasma if the free energy liberated by the instability is small compared to the total plasma energy and large compared to thermal noise.

When the energy in the excited modes is of the same order as the total plasma energy, the plasma is said to be strongly turbulent and the weak turbulence perturbation expansion fails. Since there is no satisfactory theory of strong turbulence at the present time, our consideration is limited to weak turbulence.

This theory can be discussed in terms of three basic interactions: the nonlinear wave–wave interaction, the linear (or quasilinear) wave–particle interaction, and the nonlinear wave–particle interaction. The plan of this book is to devote one chapter to each of these basic interactions, and the examples given in each chapter are chosen to illustrate some aspect of the particular interaction under consideration.

The first interaction treated is the nonlinear wave–wave interaction, which is sometimes called resonant wave–wave scattering or the decay instability. The resonance conditions for this interaction can be written as $\omega_3 = \omega_1 \pm \omega_2$ and $\mathbf{k}_3 = \mathbf{k}_1 \pm \mathbf{k}_2$, where $(\omega_1, \omega_2, \omega_3)$ and $(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ are the frequencies and wave numbers of the three waves involved in the interaction. As might be guessed from the resonance conditions, the basic mechanism behind this interaction is the strong nonlinear coupling that can occur when two waves beat together such that their sum or difference frequency and wavelength just match the frequency and wavelength of a third wave. Since the interaction does not involve resonant particles, it can be derived from fluid equations (i.e., it is not necessary to use the Vlasov equation). By interpreting ω and $\mathbf{k}$ as the energy and momentum of a single quantum associated with the kth wave, it can be seen that the resonance